

## Arithmetic

Arithmetic Sequences Notes

**Definition:** A sequence is an: *ordered set of numbers*

- Finite - set number of terms

Infinite - doesn't end

**Definition:** An arithmetic sequence is a sequence in which each term after the first is found by adding a constant, called the common difference (d) to the previous term.

**Example 1:** Fill in the next 3 terms. What is the common difference?

2, 5, 8, 11, 14, 17, 20, 23, 26       $d = 3$

**Example 2:** Fill in the next 3 terms. What is the common difference?

55, 49, 43, 37, 31, 25       $d = -6$

Term Notation: 2, 5, 8, 11, ...       $d = 3$

Term:	Symbol:	Number	(Symbolic Notation) Formula
First Term	$a_1$	$2 = 2$	$a_1$
Second Term	$a_2$	$5 = 2 + 3$	$a_1 + d$
Third Term	$a_3$	$8 = 2 + 2(3)$	$a_1 + 2d$
Fourth Term	$a_4$	$11 = 2 + 3(3)$	$a_1 + 3d$
⋮			
$n^{\text{th}}$ Term	$a_n$	$2 + (n-1)(3)$	$a_1 + (n-1)d$

Formula for writing an equation of an arithmetic sequence:

The  $n$ th term  $a_n$  of an arithmetic sequence with first term  $a_1$  and common difference  $d$  is given by:

Explicit

$$a_n = a_1 + (n-1)d$$

*n*th term →  $a_n$     first term →  $a_1$     term number →  $n$     common difference →  $d$

Where  $n$  is any positive integer. (\* $n$  stands for the number of terms in the sequence)

Example: Write an equation for the  $n$ th term of the arithmetic sequence

8, 17, 26, 35, ...

METHOD 1

Need  $a_1$  and  $d$

$$a_1 = 8$$

$$d = 9$$

$$n = n$$

$$a_n = a_n$$

$$a_n = a_1 + (n-1)d$$
$$a_n = 8 + (n-1)(9)$$

$$a_n = \underline{8} + 9n - \underline{9}$$

$$a_n = 9n - 1$$

~~METHOD 2~~

Ex: Find the 18<sup>th</sup> term

$$\uparrow \rightarrow a_{18} \rightarrow n = 18$$

$$a_{18} = 9(18) - 1 = 161$$

$$a_{18} = 161$$

Example: Use either method to find the equation for the  $n$ th term of  $-8, -6, -4, \dots$

$$a_1 = -8$$

$$d = 2$$

$$a_n = a_1 + (n-1)d$$

$$a_n = -8 + (n-1)(2)$$

$$a_n = -8 + 2n - 2$$

$$a_n = 2n - 10$$

B) Find  $a_{13}$

↳ 13<sup>th</sup> term

$$a_{13} = 2(13) - 10$$

$$a_{13} = 16$$

c) Find the 120<sup>th</sup> term.

$$a_{120} = 230$$

$$a_{120} = 2(120) - 10$$

Example: Find the 4 arithmetic means between 16 and 91. (This means the 4 missing terms between 16 and 91). \*Need to find  $d$ .

ordered pairs  
( $n, a_n$ )

$$\begin{array}{cccccc} \underline{16} & \underline{31} & \underline{46} & \underline{61} & \underline{76} & \underline{91} \\ \cdot a_1 & \times 15 & \times 15 & \times 15 & \times 15 & a_6 \\ (1, 16) & & & & & (6, 91) \end{array}$$

$$d = m = \frac{91 - 16}{6 - 1} = 15$$

$$\begin{aligned} \textcircled{2} \quad a_1 &= 16 \\ a_6 &= 91 \\ n &= 6 \end{aligned}$$

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ 91 &= 16 + (6-1)d \\ 91 &= 16 + 5d \rightarrow d = 15 \end{aligned}$$

Example: Given  $a_1 = 21$  and  $a_5 = 45$ , find the equation of the arithmetic sequence and find the 3 arithmetic means between them.

$$21, \underline{27}, \underline{33}, \underline{39}, 45$$

$(1, 21)$   $(5, 45)$

$$d = \frac{45-21}{5-1} = 6$$

$$a_n = 21 + (n-1)(6)$$

$$a_n = 6n + 15$$

Example: Given 52, 46, 40, 34... Write a formula and find the 97<sup>th</sup> term.

$$a_1 = 52$$

$$a_n = 52 + (n-1)(-6)$$

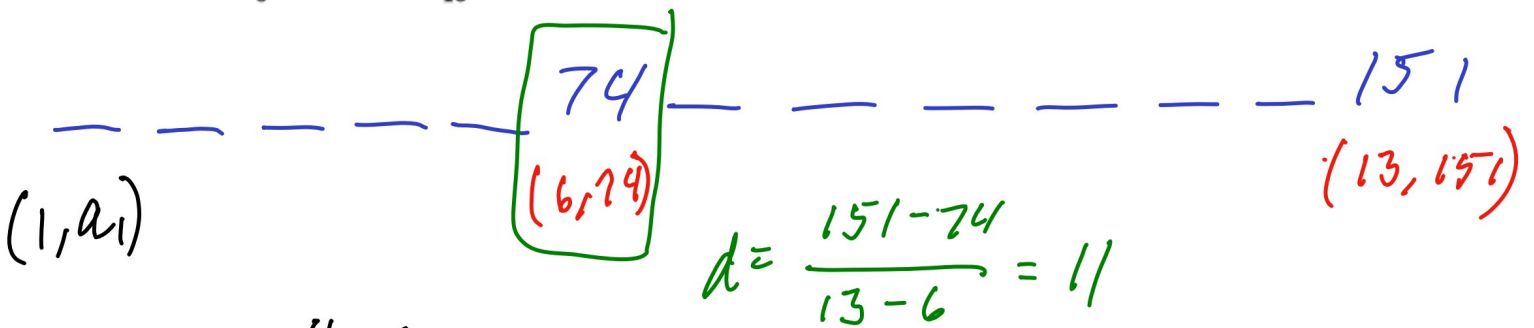
$$a_{97} = -6(97) + 58$$

$$d = -6$$

$$a_n = -6n + 58$$

$$a_{97} = -524$$

Given  $a_6 = 74$  and  $a_{13} = 151$ . Find the first term.



$$11 = \frac{74 - a_1}{6 - 1}$$

$$11 = \frac{74 - a_1}{5}$$

$$55 = 74 - a_1$$

$$a_1 = 19$$

$$a_n = a_1 + (n-1)d$$

$$74 = a_1 + (6-1)(11)$$

$$74 = a_1 + 55$$

$$a_1 = 19$$

Given the sequence:

52, 39, 26, ..., -260, -273, -286

$$a_1 = 52 \quad d = -13$$

1. Write the formula
2. Find the 12<sup>th</sup> term.
3. Which term is -182?
4. How many terms are in the sequence?

$$\begin{aligned} \textcircled{1} \quad a_n &= a_1 + (n-1)d \\ a_n &= 52 + (n-1)(-13) \\ a_n &= 52 - 13n + 13 \\ \boxed{a_n &= -13n + 65} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad a_{12} &= -13(12) + 65 \\ \boxed{a_{12} &= -91} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad n &= ? & a_n &= -13n + 65 \\ a_1 &= 52 & -182 &= -13n + 65 \\ d &= -13 & n &= 19 \\ a_n &= -182 & \boxed{a_{19} &= -182} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad a_n &= -286 \\ -286 &= -13n + 65 \\ -65 & \quad -65 \\ \boxed{n &= 27} \end{aligned}$$